Universal Fluctuation-Response Relations of Nonequilibrium Dynamics

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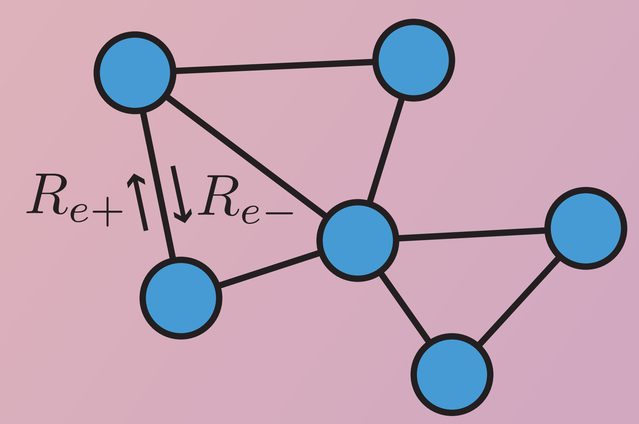
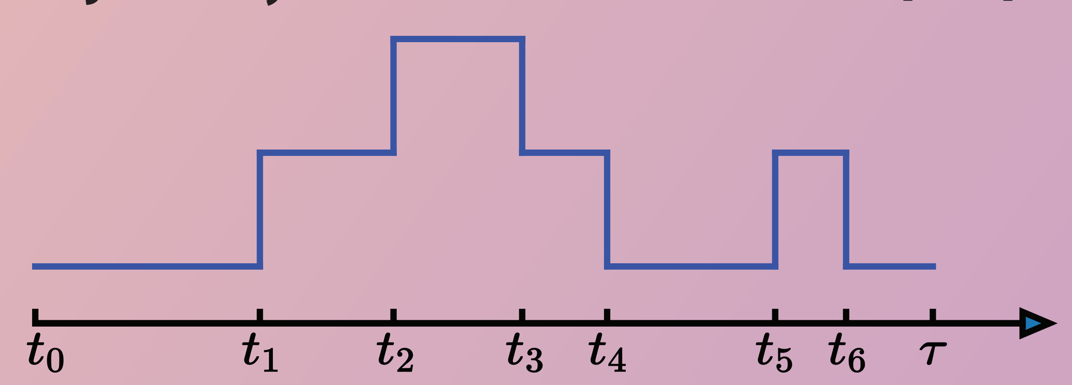
System and Setup

Markov dynamics

$$\frac{\partial \boldsymbol{p}(t)}{\partial t} = R \cdot \boldsymbol{p}(t)$$



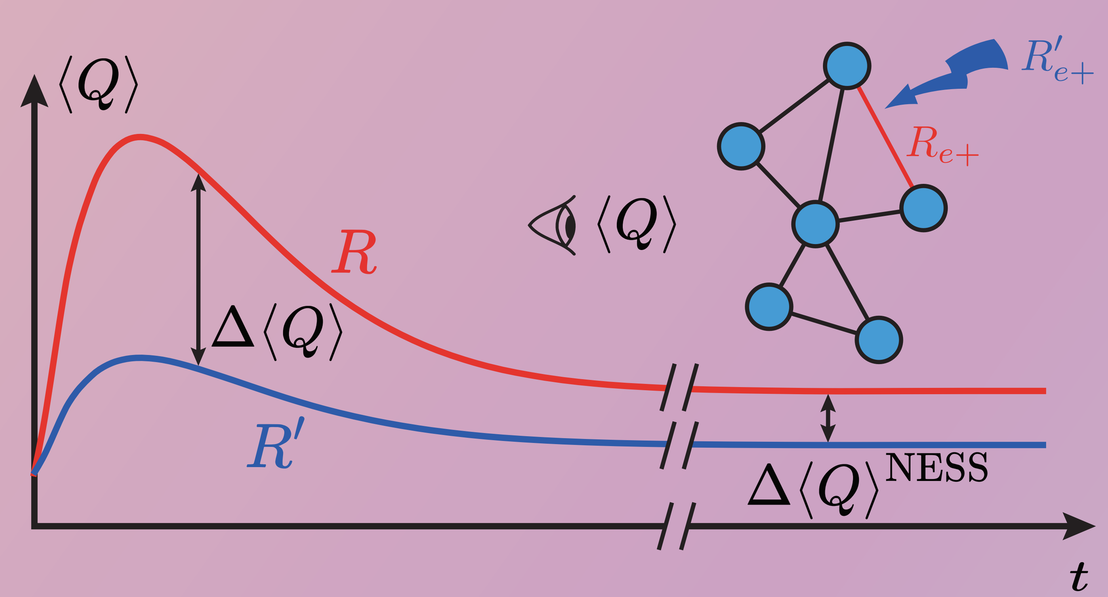
Trajectory X_τ & Observable $Q[X_\tau]$:



$X_\tau = ((x_0, t_0), (x_1, t_1), \cdots, (x_n, t_n))$

$Q = Q[X_\tau]$

Modeling non-equilibrium response



response: $\Delta\langle Q \rangle = |\langle Q \rangle' - \langle Q \rangle|$

arXiv paper

Markov dynamics can effectively describe a wide range of thermodynamic and biological processes. Understanding how such systems respond to changes in environmental variables or external inputs is crucial for predicting and controlling their behaviors. We presents universal thermodynamic bounds on the responsiveness of any Markov system toward environmental changes. The systems of interest can be arbitrarily far from stationary state. The central element of this work is the introduction of information geometry on the manifold of probability distributions of stochastic trajectories. This work lays the foundation for understanding biological processes, engineering artificial systems, and exploring the fundamental principles governing complex systems far from equilibrium.

$$|\partial_{R_{e\pm}} \langle Q \rangle| \le \sqrt{\mathrm{Var}[Q]} \cdot \frac{\sqrt{\mathcal{A}_{e\pm}}}{R_{e\pm}}$$

sensitivity relation

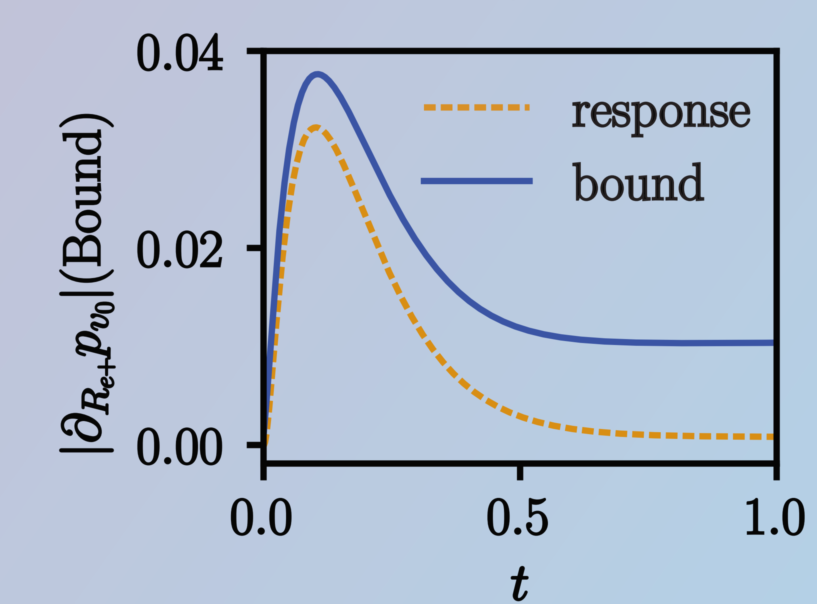
$$|\langle Q \rangle' - \langle Q \rangle| \le 2 \max_{X_\tau} |Q| \cdot \sin(G_{e\pm})$$

finite response relation

Sensitivity Relation

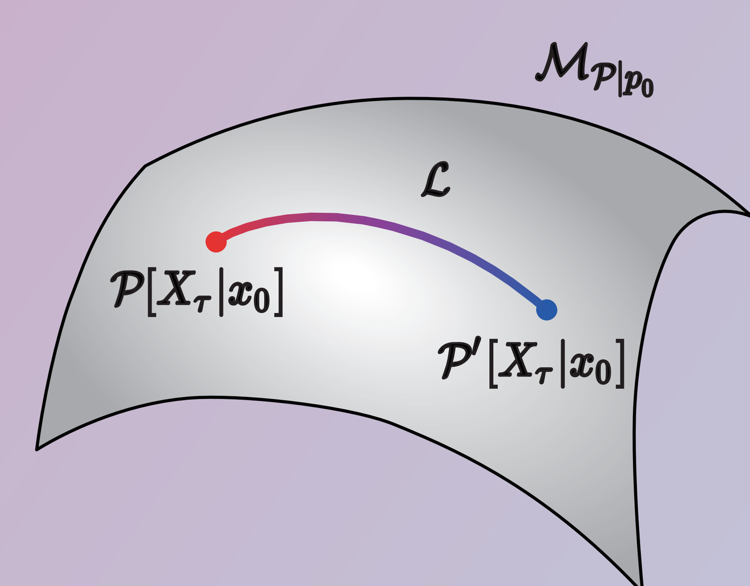
$$|\partial_{R_{e\pm}} \langle Q \rangle| \le \sqrt{\mathrm{Var}[Q]} \cdot \frac{\sqrt{\mathcal{A}_{e\pm}}}{R_{e\pm}}$$

System's sensitivity on observable Q to transition rate change is upper bounded by its variance and the dynamical activity $\mathcal{A}_{e\pm}$ (total number of transitions during the process).



capturing non-monotonic transitiant response

Finite Response Relation



Geometric bound (Information Geometry)

$$|\langle Q \rangle' - \langle Q \rangle| \le 2 \max_{X_\tau} |Q| \cdot \sin(G_{e\pm})$$

where $$G_{e\pm} = \left| \sqrt{R'_{e\pm}\tau} - \sqrt{R_{e\pm}\tau} \right|$$